Optimization

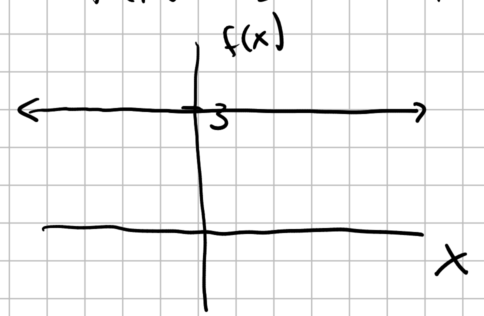
- what it is → $\boxed{\text{maxima / minima}}$ of functions → extrema (plural nouns) (functions of one variable or more)
  ↑ chapter 8 ↑ chapter 13
- where you have seen it
  → economics subjects
  → consumer theory
  → producer theory
  production function
- how to do it

→ Could have constraints / restrictions
Chapter 14 "

→ looking forward to machine learning topics (MML Book)

Example

$f(x) = 3$ Find the x's which will ~~maximize~~ minimize the value of f.



$f(x)=3$
↑
$f(0)=3, f(1)=3, \ldots$

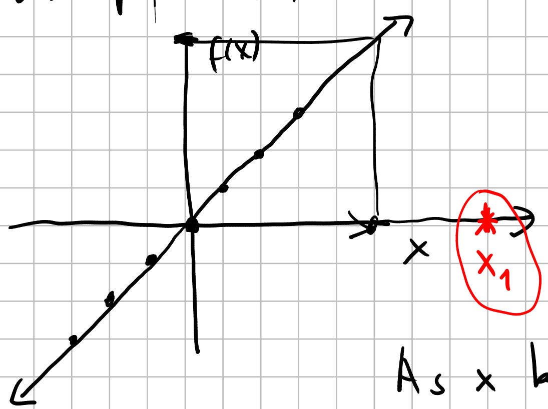
for any x, f(x) will always be equal to 3.

→ Any value of x will ~~maximize~~ minimize f.

⇒ infinite number of ~~maximizers~~ minimizers.

Example

$f(x) = x$ → Linear function.



x	f(x)
0	$f(0)=0$
1	$f(1)=1$
⋮	⋮

* Find the x's which maximize f.

→ There is no maximizer for f.

NOTE infinity is not a number
→ "concept"

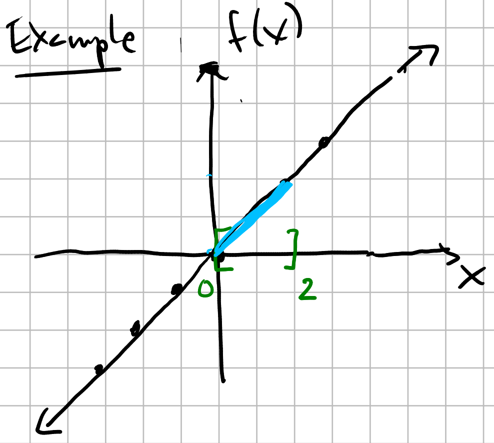
As x becomes large, f(x) also becomes large.

$$\lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to -\infty} f(x) = -\infty$$

If you say x_1 is a maximizer of f, you will find that there is another value of x, namely x_1+1 (for example), which will give a larger value of f.

$$f(x_1+1) = x_1+1 > x_1$$

Example



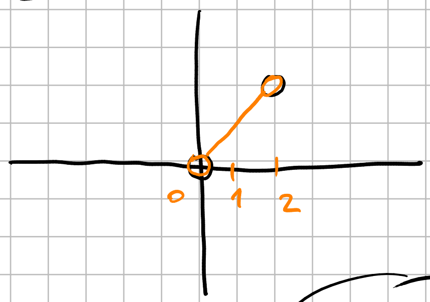
$f(x) = x$

What could we do to at least guarantee that there would be a maximizer / a minimizer of f?

Find the x's which will ~~maximize~~ minimize f over $x \in$ ~~[0,2]~~ $(0,2)$.

restriction/constraint

$\Rightarrow x = 2$ maximizes f over $x \in [0,2]$.



$x \in [0,2] \Leftrightarrow 0 \leq x \leq 2$

$x \in (0,2) \Leftrightarrow 0 < x < 2$

$\Rightarrow x = 0$ minimizes f over $x \in [0,2]$

1,000001

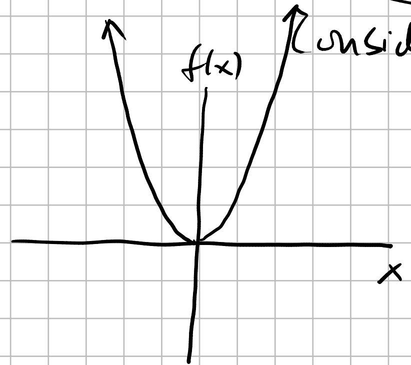
$[0, 2]$ attain/reach $x=0, x=2$

$(0,2)$ approach $x=0, x=2$

0.0000005

0.000000005

Example 1 (a) $f(x) = 3 - (x-2)^2$



Consider $f(x) = x^2$

x	$f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Here $x = 0$ is a minimizer of $f(x) = x^2$.

There is no maximizer of f.

maximizer / minimizer

max value / min value