Conditions

- f is differentiable on some interval I
- c is an interior point of I
- c is an extreme point of f *

$\Rightarrow$ $f'(c) = 0$. → necessary condition

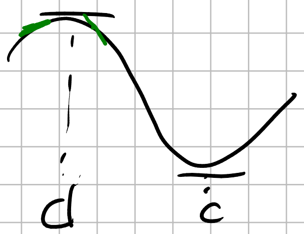
(geometric sense: the slope of the tangent line to f at c is equal to zero (horizontal tangent line at c))

- Not very useful to verify if you have max/min.
- Very useful to find potential candidates for max/min. Do checking afterwards.
- Very useful to rule out purported max/min.

$$c \text{ is an extreme pt of } f \Rightarrow f'(c) = 0$$

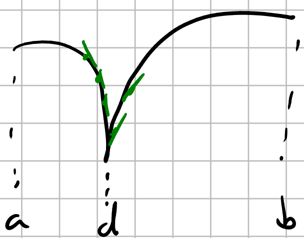
Logic equivalent statement:

$$f'(c) \neq 0 \Rightarrow c \text{ is not an extreme pt of } f.$$



related to Fig 2 Section 8.1

Figure 3



d is a minim pt of f over $[a,b]$

but $f'(d)$ does not exist

(f is not differentiable at d)

differentiable (visually smooth or no sharp turns)

sudden changes in the slope of the function → indicative of non differentiability

Section 8.2

Example 1

$$c(t) = \frac{t}{t^2+4}$$

Step 1 $\quad c'(t) = \cdots = \dfrac{(2+t)(2-t)}{(t^2+4)^2}$

Step 2 stationary pts (candidates for extreme pts)

values of t for which $c'(t)=0$

$$\frac{(2+t)(2-t)}{(t^2+4)^2}=0 \Rightarrow (2+t)(2-t)=0$$

$$\Rightarrow 2+t=0 \text{ or } 2-t=0$$

$t=-2$ ruled out by $t\geq 0$ $\qquad t=2$

The only candidate extreme pt is $t=2$.

Step 3. When is $c'(t)\geq 0$? When is $c'(t)\leq 0$?

When $0\leq t<2$, $c'(t)=\frac{(2+t)(2-t)}{(4+t^2)^2}=\frac{(+)(+)}{(+)}>0$

When $t>2$, $c'(t)=\frac{(2+t)(2-t)}{(4+t^2)^2}=\frac{(+)(-)}{(+)}<0$

By the first derivative test, $t=2$ is a maximum point of $c(t)$.

NOTE Why is there $\boxed{0\leq t}<2$? ① given in the problem. ② More importantly, $2+t$ could become negative if we do not have $t\geq 0$.

Section 8.2 Example 2

Zeros of $f(x)$: values of x such that $f(x)=0$.

e is a constant

$f(x)=\boxed{e^{2x}}-\boxed{5e^x}+\boxed{4}$

$\frac{d}{dx}[e^x]=e^x$ does not depend on x

do not have exactly the same pattern

$f'(x)=\frac{d}{dx}[e^{2x}]+\frac{d}{dx}[-5e^x]+\frac{d}{dx}[4]$

$\frac{d}{dx}[c]=0$ (c is a constant)

chain rule

$=e^{2x}\cdot 2 \quad -5\frac{d}{dx}[e^x]+0$

$=2e^{2x} \quad -5e^x$

$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$

$\frac{d}{dx}[cf(x)]=c\frac{d}{dx}[f(x)]$ (c is a constant)

$\frac{d}{dx}[f(g(x))]=f'(g(x))\,g'(x)$

$\frac{d}{dx}[x^n]=nx^{n-1}$ (n is a constant)

$f'(x) = 2e^{2x} - 5e^{x}$ You have to fill in the details

Do like what you see in the book or.

$(e^x)^2$

$f'(x) = 0 \Rightarrow 2e^{2x} - 5e^x = 0$

$\Rightarrow \underline{e^x}(\underline{2e^x - 5}) = 0$

But e^x is never zero regardless of the value of x, so $2e^x - 5 = 0$

$2e^x = 5$

$e^x = 5/2$

$\log e^x = \log \frac{5}{2}$

$x \log e = \log 5/2$

$x(1) = \log 5/2$

$x = \log(5/2)$

mentioned in Lab!

$\log = \ln$ in economics

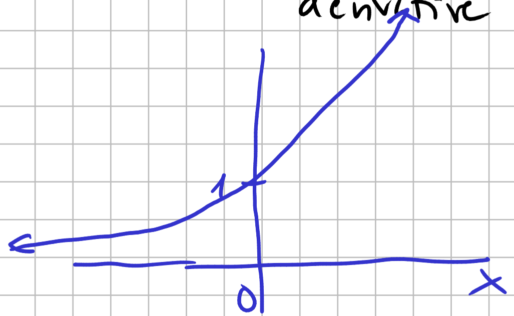
When $x < \log(\frac{5}{2})$, $f'(x) = \boxed{e^x}(\underline{2e^x - 5}) = (+)(-) < 0$

try $x=0$

When $\underline{x > \log(\frac{5}{2})}$, $f'(x) = \underline{e^x}(\underline{2e^x - 5}) = (+)(+) > 0$

try $x = 10$

Therefore, $x = \log(5/2)$ is a minimum point by the first derivative test.



e^x

(c) $f(x) = (\boxed{e^x} - 1)(e^x - 4)$

$\lim_{x \to -\infty} f(x) = (0-1)(0-4) = 4$

$(f')'(x) \leq 0$

f is concave $\Leftrightarrow$ $f''(x) \leq 0$ for all x in I

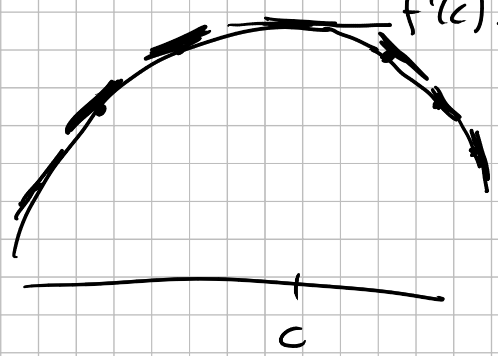
The second derivative is the derivative of f' first

$(f')'$ slope of f'

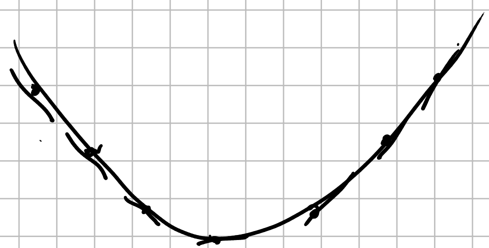
$(f')'(x) \leq 0 \Leftrightarrow$ slope of f' is negative

$\Leftrightarrow f'$ is decreasing

$f'(c) = 0$



$f''(x) \leq 0 \Leftrightarrow f'$ is decreasing.



It may seem that Thm 8.2.2 is easier to apply than First Derivative Test. But your mileage may vary.

Go back to Section 8.2 Example 1.

$c(t) = \dfrac{t}{t^2+4}$ $\quad c'(t) = \dfrac{(2+t)(2-t)}{(t^2+4)^2}$

$c''(t) =$

This could be a lot of work (prone to mistakes!)

Section 8.2 Exercise 3

$h(t) = \sqrt{t} - \frac{1}{2}t \quad t \in [0,3]$ $\quad \sqrt{t} \to t^{1/2}$

$h'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2}$

Find stationary pts: $h'(t) = 0 \Rightarrow \frac{1}{2}t^{-1/2} - \frac{1}{2} = 0$

$\Rightarrow t^{-1/2} = 1$

$$\frac{1}{t^{1/2}} = 1 \Rightarrow (1 = t^{1/2})^2 \Rightarrow t = 1.$$

Two options $\quad t \in [0,3]$ $\quad$ 0 & 3 are not interior points

Option 1

when $0 \le t < 1$, $h'(t) = \left(\frac{1}{2}t^{-1/2} - \frac{1}{2}\right) > 0$

try $t = 0.5$ (do not try $t=0$!)

when $1 < t \le 3$, $h'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2} < 0$

try $t = 2$

By the first derivative test, $t = 2$ is the max pt of h.

$\Rightarrow$ It takes 2 months for the plant to be at its tallest.

Option 2

$t^{3/2} = \sqrt{t^3}$ or $(\sqrt{t})^3$

You know that $h'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2}$

$\Rightarrow h''(t) = -\frac{1}{4}t^{-3/2} = -\frac{1}{4t^{3/2}} = \frac{-1}{4(t)}$

when $t \in [0,3]$, $h''(t) < 0$.

Therefore, h is a concave function of t.

$\Rightarrow$ It takes 2 months for the plant to be at its tallest.

Section 8.3 Example 1

profits = total revenue − total costs $\quad Y(N)$

= price * quantity − costs.

$N \to$ pounds of fertilizer

$Y \to$ bushels of wheat

$$\pi(N) = 1.4(-13.62 + 0.984N - 0.05N^{1.5}) - 0.18N$$

Group HW tomorrow at Neocities

groupings available at Animospace tomorrow