$$c'(t) = \frac{(2+t)(2-t)}{(t^2+4)^2}$$

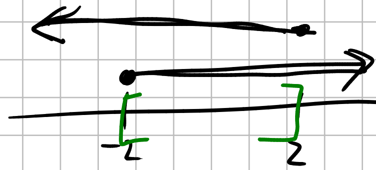
whether or not $c'(t) \geq 0$

Since $(t^2+4)^2 \geq 0$, to have $c'(t) \geq 0$ we must have

$$\boxed{(2+t)(2-t) \geq 0}$$

Both $2+t \geq 0$ & $2-t \geq 0$ ✓

$t \geq -2$ & $-t \geq -2 \Rightarrow t \leq 2$

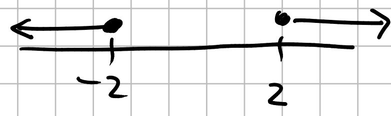


$\Rightarrow -2 \leq t \leq 2$

OR

Both $2+t \leq 0$ & $2-t \leq 0$ ✓

$t \leq -2$ & $-t \leq -2 \Rightarrow t \geq 2$



no t will satisfy both ineq.

To ensure that $c'(t) \geq 0$, we must have $-2 \leq t \leq 2$.

$$c''(t) = \frac{\overset{(+)}{\boxed{2t}}\boxed{(t^2-12)}}{\boxed{(t^2+4)^3}_{(+)}}$$

whether or not $c(t)$ is concave/convex over $t \geq 0$

Try $t = 1 \Rightarrow t^2 - 12 < 0$

Try $t = 4 \Rightarrow t^2 - 12 > 0$

When will $c(t)$ be concave?

$\Rightarrow$ when $t^2 - 12 \leq 0$

$t^2 = 12$

$t = \pm\sqrt{12} = \pm 2\sqrt{3}$

only $+2\sqrt{3}$ or $+\sqrt{12}$

When $0 \leq t \leq \sqrt{12}$, $t^2 - 12 \leq 0$. $c(t)$ concave when $0 \leq t \leq \sqrt{12}$

When $t \geq \sqrt{12}$, $t^2 - 12 \geq 0$. $c(t)$ convex when $t \geq \sqrt{12}$.

$N \rightarrow N+1$

$PY(N) \longrightarrow PY(N+1)$

exact change

diff $= PY(N+1) - PY(N)$

approximate change

$\approx PY'(N)$

Options available
→ First Derivative test
→ check concavity/convexity (2nd derivative)

$D = \mathbb{R} \rightsquigarrow D = [5, \infty) \rightsquigarrow D = (0, \infty)$
$\rightsquigarrow D = [0, +\infty)$

} settings encountered so far

# of stationary pts = 1 $\rightsquigarrow$ none.

Section 8.3 Example 3 (no functional forms)

constant

$$\pi(Q) = Q\,P(Q) - kQ$$

$$\pi'(Q) = Q\,P'(Q) + P(Q)\cdot 1 - k\cdot 1$$
$$= Q\,P'(Q) + P(Q) - k$$

$\frac{d}{dx}[x] = 1$

$\frac{d}{dx}[f(x)g(x)]$
$= f(x)g'(x) + g(x)f'(x)$

(b) $\frac{dQ^*}{dk} = ?$ You do not know what Q^* looks like!

You only know that $Q^* > 0$ satisfies $\pi'(Q^*) = 0$.

$$Q^* P'(Q^*) + P(Q^*) - k = 0$$

$\vdots$

Q^* = depend on k and other features of $P(Q)$

Think of Q^* as a qty that depends on k

$$Q^*(k)\,P'(Q^*(k)) + P(Q^*(k)) - k = 0$$

Take derivative of FOC wrt k.
(implicit differentiation)

$$\frac{d}{dk}\left[Q^*(k)\,P'(Q^*(k)) + P(Q^*(k)) - k\right] = \underbrace{\frac{d}{dk}[0]}_{0}.$$

$$Q^*(k)\,P''(Q^*(k))\boxed{\frac{dQ^*}{dk}} + P'(Q^*(k))\boxed{\frac{dQ^*}{dk}} + P'(Q^*(k))\boxed{\frac{dQ^*}{dk}} - 1 = 0$$

Solve for $\frac{dQ^*}{dk}$!

$$\frac{dQ^*}{dk}\left[Q^*(k)\,P''(Q^*(k)) + \underbrace{P'(Q^*(k)) + P'(Q^*(k))}\right] = 1$$

$$\frac{dQ^*}{dk} = \frac{1}{Q^*(k)\,P''(Q^*(k)) + 2P'(Q^*(k))}$$

In the majors, you will be asked to use economics to give the sign of this quantity!

(c) You don't know what Q^* looks like!

optimal profit $= \pi(Q^*) = Q^* P(Q^*) - k Q^*$

$= Q^*(k) P(Q^*(k)) - k Q^*(k)$

$$\frac{d\pi(Q^*)}{dk} = ?$$

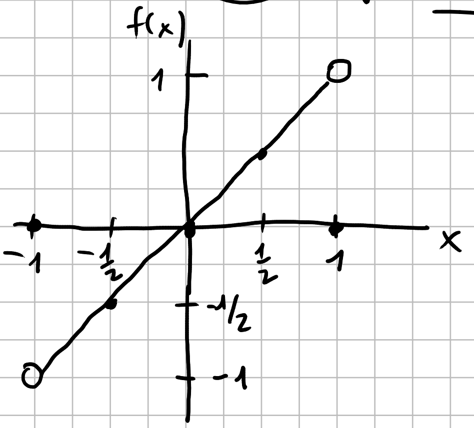
Section 8.4 sufficient conditions to guarantee existence of max/min.

practical → in order for us to proceed in looking for them

theoretical →

Section 8.4 Exercise 8

$$f(x) = \begin{cases} x & \text{if } x \in (-1, 1) \\ 0 & \text{if } x = -1 \text{ or } x = 1 \end{cases}$$



max/min ?