Chapter ~~8~~ 13 optimization for functions of ~~a single (one) variable~~ two or more variables

Determine which value(s) of x will $f(x)$ be at its largest / smallest.

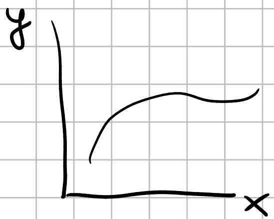
(x, y) $f(x, y)$

economic agent $(x_1, x_2, \ldots, x_n)$ $f(x_1, x_2, \ldots, x_n)$

extend to more realistic situations
- consumer choose many goods to max utility
- firms choose input levels (capital, labor) to max profits / min costs
- firms choose how much to produce of each good to max profits / min costs

Chapter 13 vs Chapter 8 : Pay attention to what things you have (not) seen in Chapter 8 that don't (do) appear in Chapter 13

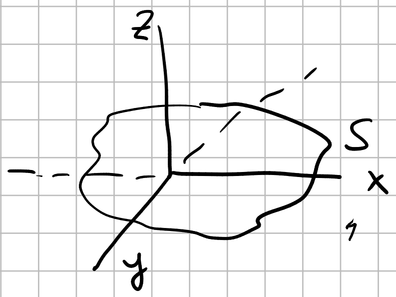
$y = f(x)$

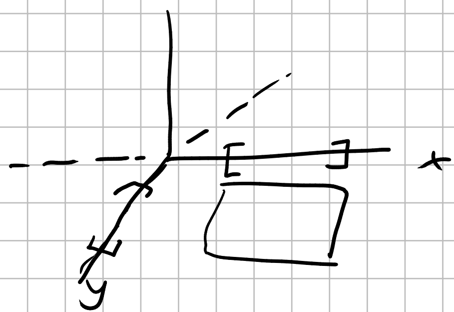


$w = f(x, y, z)$

could not visualize this anymore!

$z = f(x, y)$



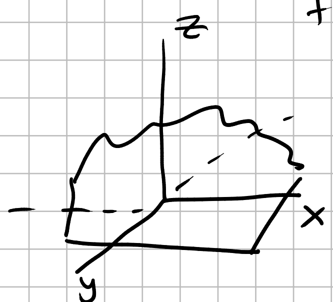


maximize / minimize $f(x)$ on $I = [a, b]$ ← example

I ↓ interval

where x's come from.

$f(x, y)$ on S (on the xy plane)



fixed

✓ $g(x) = f(x, y_0)$ function of x alone

✓ $h(y) = f(x_0, y)$ function of y alone.

fixed

P
If $(x_0, y_0, f(x_0, y_0))$ is the highest point, $g'(x)$ evaluate it at $x = x_0$ $\Rightarrow g'(x_0) = 0$

$h'(y)$ evaluate it at $y = y_0$ $\rightarrow h'(y_0) = 0$.

partial derivative!

take the derivative

$$g'(x) = \boxed{f_1^{\prime}(x, y_0)} \qquad \frac{\partial f}{\partial x}(x, y_0) \sim \frac{\partial f(x, y_0)}{\partial x} \sim f_1(x, y_0)$$

with respect to which argument?

the book uses this notation

$$h'(y) = f_2'(x_0, y) \qquad \frac{\partial f}{\partial y}(x_0, y) \sim \frac{\partial f(x_0, y)}{\partial y} \sim f_2(x_0, y)$$

Notice that necessary conditions involved first order partial derivatives at the optimum and they are both equal to zero.

$$z = -2x^2 - 2y^2$$

$$z = -2 \Rightarrow \quad -2 = -2x^2 - 2y^2$$
$$1 = x^2 + y^2 \longrightarrow$$ equation of a circle. center at origin $(0,0)$ radius 1

Fixing $x = x_0$ in $z = f(x, y)$ gives a trace / profile
Fixing $y = y_0$ in $z = f(x, y)$

Fix $z = z_0$ in $z = f(x, y) \Rightarrow f(x, y) = z_0$ ← fixed number
(implicit function) produces level curves.

Need algebra skills:

$$-4x - 2y + 36 = 0$$
$$-2x - 4y + 42 = 0$$
} System of 2 linear equations in 2 unknowns (x, y)

$$-2x - y + 18 = 0 \Rightarrow y = -2x + 18$$
$$-x - 2y + 21 = 0 \Rightarrow$$

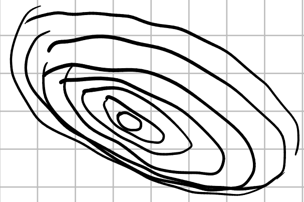
Subst

$$-x - 2(-2x + 18) + 21 = 0$$
$$-x + 4x - 36 + 21 = 0$$
$$3x = 15$$
$$x = 5$$
$$y = -2(5) + 18 = 8$$

elim

$$\boxed{-2x - y + 18 = 0}$$
$$(-2)(-x - 2y + 21 = 0)$$
$$\boxed{2x + 4y - 42 = 0}$$

add both in boxes $\Rightarrow 3y - 24 = 0 \Rightarrow y = 8$



concentric ellipses

K^*, L^* optimum level of inputs which will maximize profits

$\Rightarrow \quad pF_K'(K^*,L^*)=r \quad \& \quad pF_L'(K^*,L^*)=w$

$\Rightarrow \quad \underline{F_K'(K^*,L^*)} = \underline{\frac{r}{p}} \quad \& \quad \underline{F_L'(K^*,L^*)} = \underline{\frac{w}{p}}$

marginal product of capital

marginal product of labor

wage rate in terms of price of good

price of labor relative to price of good

(real wage)

$\cancel{\sqrt{x+b} = x^{\frac{1}{2}} + b^{\frac{1}{2}}}$

$z = f(x,y)$

$\boxed{\frac{\partial f}{\partial x}}, \frac{\partial f}{\partial y}$

$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$

$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$

$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$