Section 13.5 Example 1

$b_1 > 0$
& $b_2 > 0$ } otherwise, no maximum

demand curves slope downward

$\alpha > 0$ costlier to make more

to guarantee that $Q_1^*, Q_2^* > 0$, we must have $a_1 > \alpha$, $a_2 > \alpha$.

$$P_1^* = \underbrace{\tfrac{1}{2}(a_1 + \alpha)}\overset{\text{because } a_1 > \alpha}{>}\underbrace{\tfrac{1}{2}(\alpha + \alpha)} = \tfrac{1}{2}(2\alpha) = \alpha$$

$P_2^* > \alpha$ $(a_2 > \alpha)$

"dumping" → international trade

$P_1 = a_1 - b_1 Q_1$ Ex 1

$P_1 = 100 - Q_1$ Ex 2

$a_1 = 100$, $b_1 = 1$

$P_1 = 100 - Q_1$ demand

$P_2 = 80 - Q_2$ demand

$P_1 = P_2 = P$ (don't confuse this with mkt equilibrium)

$P = 100 - Q_1 \Rightarrow Q_1 = 100 - P$

$P = 80 - Q_2 \Rightarrow Q_2 = 80 - P$

$Q = Q_1 + Q_2 = 180 - 2P$

$P = 90 - \frac{1}{2}Q$

just equal

$47t - \frac{1}{4}t^2$ fall in profits

$47t - \frac{1}{2}t^2$ gain in taxes.

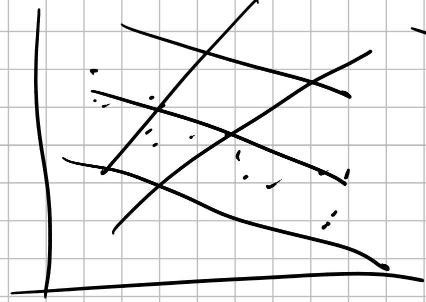
Example 4 is meant to be illustrative only.

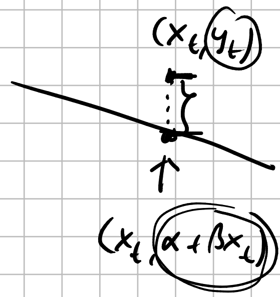
→ give an idea of what linear regression is

→ in reality, the example is a toy.

t	x_t	y_t	$y_t - (\alpha + \beta x_t)$
1	x_1	y_1	$y_1 - \alpha - \beta x_1$
2	x_2	y_2	$y_2 - \alpha - \beta x_2$
3	x_3	y_3	$y_3 - \alpha - \beta x_3$
4	⋮	⋮	⋮
⋮			
T		y_T	

= Sum( )





"gaps" / "discrepancies" from the line being fit

linear regression →

$$(y_1 - \alpha - \beta x_1)^2 \quad "$$
$$+ (y_2 - \alpha - \beta x_2)^2 \quad "$$
$$\vdots$$
$$+ (y_T - \alpha - \beta x_T)^2 \quad "$$

$$(y_1 - \alpha - \beta x_1)^2 + (y_2 - \alpha - \beta x_2)^2 + \dots + (y_T - \alpha - \beta x_T)^2$$

$$= \sum_{\boxed{t=1}}^{\boxed{T}} (y_t - \alpha - \beta x_t)^2$$

↳ Summation symbol

Linear regression: Task is to find α, β so that $\sum_{t=1}^{T} (y_t - \alpha - \beta x_t)^2$ is minimized.

$$\min_{\alpha, \beta} \underbrace{(y_1 - \alpha - \beta x_1)^2 + (y_2 - \alpha - \beta x_2)^2 + \dots + (y_T - \alpha - \beta x_T)^2}_{L(\alpha, \beta)}$$

$$\frac{\partial L(\alpha, \beta)}{\partial \alpha} = 2(y_1 - \alpha - \beta x_1)(-1) + 2(y_2 - \alpha - \beta x_2)(-1) + \dots + 2(y_T - \alpha - \beta x_T)(-1)$$

$$\frac{\partial L(\alpha,\beta)}{\partial \beta} = 2(y_1-\alpha-\beta x_1)(-x_1) + 2(y_2-\alpha-\beta x_2)(-x_2) + \dots + 2(y_T-\alpha-\beta x_T)(-x_T)$$

$$2(y_1-\alpha-\beta x_1)(-1) + 2(y_2-\alpha-\beta x_2)(-1) + \dots + 2(y_T-\alpha-\beta x_T)(-1) = 0$$

$$2(y_1-\alpha-\beta x_1)(-x_1) + 2(y_2-\alpha-\beta x_2)(-x_2) + \dots + 2(y_T-\alpha-\beta x_T)(-x_T) = 0$$

$$y_1 - \alpha - \beta x_1 + y_2 - \alpha - \beta x_2 + \dots + y_T - \alpha - \beta x_T = 0$$

$$(y_1-\alpha-\beta x_1)x_1 + (y_2-\alpha-\beta x_2)x_2 + \dots + (y_T-\alpha-\beta x_T)x_T = 0$$

$$y_1 + y_2 + \dots + y_T - T\alpha - \beta(x_1 + x_2 + \dots + x_T) = 0$$

$$y_1 + y_2 + \dots + y_T - \beta(x_1 + x_2 + \dots + x_T) = T\alpha$$

$$\frac{y_1 + y_2 + \dots + y_T}{T} - \beta\left(\frac{x_1 + x_2 + \dots + x_T}{T}\right) = \alpha$$

NOTE: Book uses different notation that has its own "baggage"!

μ (mu)
σ (sigma) }

$$\underset{\text{(average of y's)}}{\bar{y}} - \beta \underset{\text{(average of x's)}}{\bar{x}} = \alpha$$

$$(y_1-\alpha-\beta x_1)x_1 + (y_2-\alpha-\beta x_2)x_2 + \dots + (y_T-\alpha-\beta x_T)x_T = 0$$

$$(y_1 - \bar{y} + \underbrace{\beta\bar{x} - \beta x_1}_{-\beta(x_1-\bar{x})})x_1 + (y_2 - \bar{y} + \beta\bar{x} - \beta x_2)x_2 + \dots + (y_T - \bar{y} + \beta\bar{x} - \beta x_T)x_T = 0$$

Solve for β.

$$(y_1-\bar{y})x_1 + (y_2-\bar{y})x_2 + \dots + (y_T-\bar{y})x_T - \beta(x_1-\bar{x})x_1 - \beta(x_2-\bar{x})x_2 - \dots - \beta(x_T-\bar{x})(x_T) = 0$$

$$\beta = \frac{(y_1-\bar{y})x_1 + (y_2-\bar{y})x_2 + \dots + (y_T-\bar{y})x_T}{(x_1-\bar{x})x_1 + (x_2-\bar{x})x_2 + \dots + (x_T-\bar{x})x_T}$$

add / subtract the same thing

$$= \frac{(y_1-\bar{y})(x_1-\bar{x}+\boxed{\bar{x}}) + (y_2-\bar{y})(x_2-\bar{x}+\boxed{\bar{x}}) + \dots + (y_T-\bar{y})(x_T-\bar{x}+\boxed{\bar{x}})}{(x_1-\bar{x})(x_1-\bar{x}+\boxed{\bar{x}}) + (x_2-\bar{x})(x_2-\bar{x}+\boxed{\bar{x}}) + \dots + (x_T-\bar{x})(x_T-\bar{x}+\boxed{\bar{x}})}$$

after some algebra

$$= \frac{(y_1-\bar{y})(x_1-\bar{x}) + \dots + (y_T-\bar{y})(x_T-\bar{x})}{(x_1-\bar{x})^2 + \dots + (x_T-\bar{x})^2}$$

$$= \frac{\frac{1}{T}\sum_{t=1}^{T}(x_t-\bar{x})(y_t-\bar{y})}{\frac{1}{T}\sum_{t=1}^{T}(x_t-\bar{x})^2}$$

t	x_t	y_t	$(x_t-\bar{x})^2$	$(x_t-\bar{x})(y_t-\bar{y})$
1				
2				
3				
4				
⋮				
T				
	$\bar{x}$	$\bar{y}$	sum / avg	sum / avg.