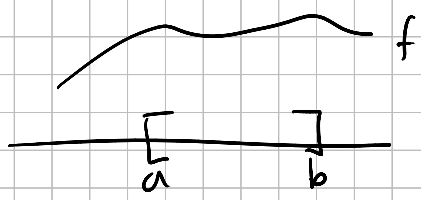
Ch 8 $S = [a,b]$

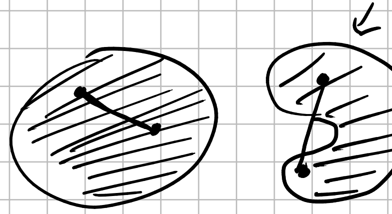


interior of $S = (a,b)$

boundary of $S = \{a, b\}$ endpoints of the interval

Ch 13 a bit more complicated:
meaning of interior, boundary?

comparison with Section 13.2 ↳ S convex set

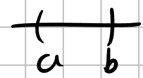


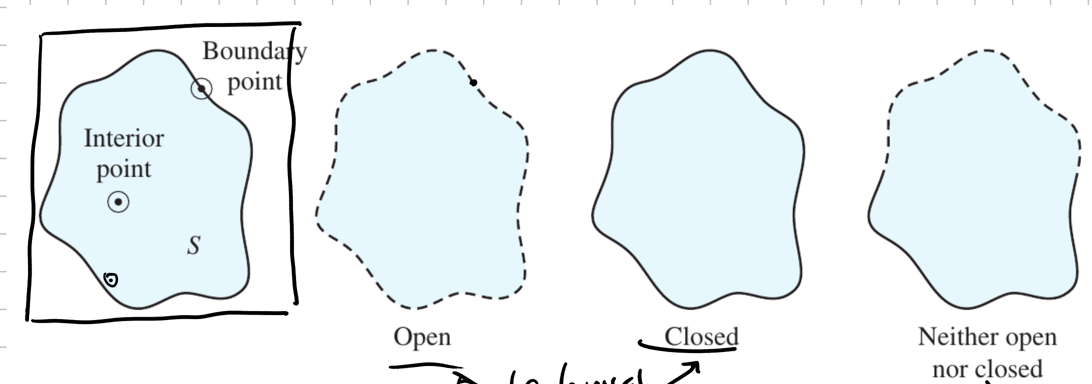
convex set

not a convex set

(x_0, y_0) $\underbrace{(x_0+h, y_0+k)}$
perturbation of (x_0, y_0)

Section 13.5 ↳ S closed & bounded set





technical

all sets in this figure are bounded

open is not the opposite of closed

In economics, typically you will encounter closed & bounded sets.

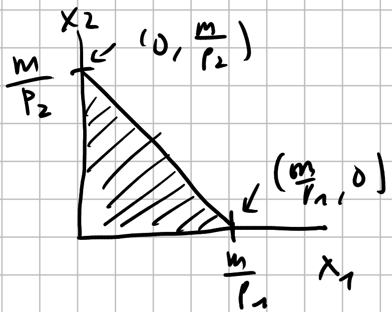
e.g. consumer choice problems

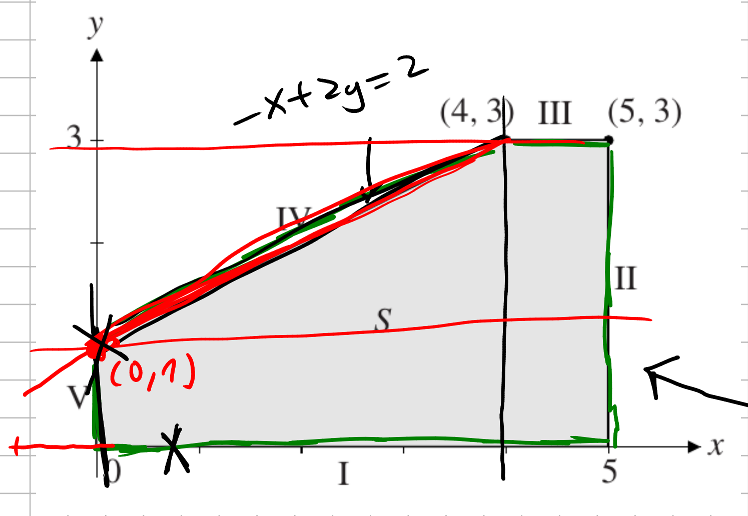
budget constraint $p_1 x_1 + p_2 x_2 \le m$ $\quad x_1, x_2 \geq 0$, $p_1, p_2 > 0$

prices

income

qty of goods you want to buy





closed & bounded set

$$f(x,y) = 9x+8y-6(x+y)^2$$

$$\max_{(x,y)\in S} f(x,y)$$

Step 1 Find stationary points of f in the interior of S.

$$\frac{\partial f}{\partial x} = 9 - 12(x+y)(1)$$
$$\frac{\partial f}{\partial y} = 8 - 12(x+y)(1)$$

$$9-12x-12y=0$$
$$8-12x-12y=0$$

$$12x+12y=9$$
$$12x+12y=8$$

These two equations do not have a solution because the two equations cannot be true simultaneously.

There are no stationary points of f in the interior of S.

Step 2

boundary represented by $\mathrm{I} = \{(x,y):\ 0\le x\le 5,\ y=0\}$
$= \{(x,0):\ 0\le x\le 5\}$

$f(x,0)$
Back to Ch 8! You have $f(x,0)$ function of x alone
$0\le x\le 5$ closed & bounded
So EVT applies!

boundary represented by $\mathrm{II} = \{(5,y):\ 0\le y\le 3\}$

boundary represented by $\mathrm{III} = \{(x,3):\ 4\le x\le 5\}$

$\mathrm{IV} = \{(x,y):\ -x+2y=2,\ 0\le x\le 4\}$ //

$= \{(x,y):\ -x+2y=2,\ 1\le y\le 3\}$ //

$\mathrm{V} = \{(0,y):\ 0\le y\le 1\}$

$\mathrm{I}:\ \max_{0\le x\le 5} f(x,0)$

$f(x,0) = 9x+8(0)-6(x+0)^2$
$= 9x-6x^2$ function of one variable

Apply recipe from Ch 8.

- Find stationary pts in the interior!

$9 - 12x = 0 \Rightarrow x = \underline{3/4}$ belongs to $0 \le x \le 5$

- Boundary points?

$\underline{x=0}, \quad \underline{x=5}$

- Test the points which give the largest value of $f(x,0)$

$f(0,0) = 9(0) + 8(0) - 6(0+0)^2 = 0$

$f(\frac{3}{4},0) = 9(\frac{3}{4}) + 8(0) - 6(\frac{3}{4}+0)^2 = \frac{27}{4} - 6(\frac{9}{16}) = \frac{27}{4} - \frac{27}{8} = \boxed{\frac{27}{8}}$

$f(5,0) = 9(5) + 8(0) - 6(5+0)^2$
$= 45 - 6(25)$
$= -105$

So, $(\frac{3}{4},0)$ gives the largest value of $f(x,y)$ along the boundary represented by I.

II: $\max_{0 \le y \le 3} f(5,y)$ $\qquad f(5,y) = 9(5) + 8y - 6(5+y)^2$
$= 45 + 8y - 6(5+y)^2$

Apply recipe! Do this for yourself.

What is the point (x,y) giving the largest value of $f(x,y)$ along boundary represented by II?

III, V $\to$ repeat the idea, do for yourself.

IV: $\max_{\substack{-x+2y=2 \\ 0 \le x \le 4}} f(x,y) = \max_{\substack{-x+2y=2 \\ 0 \le x \le 4}} 9x + 8y - 6(x+y)^2$

$2y = 2 + x$

$y = \frac{2+x}{2} = 1 + \frac{1}{2}x$

$= \max_{0 \le x \le 4} \underbrace{9x + 8(1+\frac{1}{2}x) - 6(x+1+\frac{1}{2}x)^2}_{\text{Simplify}}$

continue for yourself!

alternatively: $\max_{\substack{-x+2y=2 \\ 1 \le y \le 3}} f(x,y) = \max_{\substack{-x+2y=2 \\ 1 \le y \le 3}} 9x + 8y - 6(x+y)^2$ $\to x = 2y - 2$

$= \max_{1 \le y \le 3} \underbrace{9(2y-2) + 8y - 6(\overbrace{2y-2+y}^{3y-2})^2}$

$18y - 18 + 8y - 6(9y^2 - 12y + 4)$
$= 26y - 18 - 54y^2 + 72y - 24$

$= 98y - 42 - 54y^2$

Find stationary pts

$\Rightarrow \quad 98 - 108y = 0$

$y = \frac{98}{108}$ <1 is not in $[1,3]$

no stationary points in the interior of $[1,3]$

Boundary pts: $y=1$, $y=3$

$98(1)-42-54(1)^2 = 2$

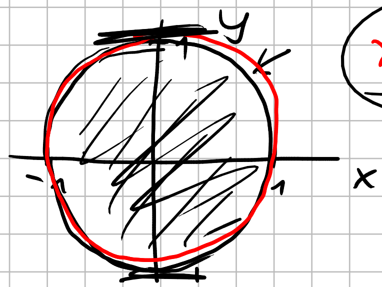
$98(3)-42-54(3)^2 = -234$

What is the point (x, y) giving the largest value of $f(x,y)$ on boundary represented by IV ?

$(0, 1)$

$x = 2y-2 = 2(1)-2$

Section 13.5 Example 1



$x^2+y^2=1$. boundary

$f(x,y) = x^2+y^2+y-1$

Incorporate boundary into function

$-1 \le y \le 1$ ← either have $g(y) = 1-y^2+y^2+y-1 = y$

OR replace y^2 with $y^2=1-x^2$

the book avoids this completely ←

$h(x) = x^2 + (1-x^2) + \bigcirc - 1$

$y^2 = 1-x^2 \Rightarrow y = \pm\sqrt{1-x^2}$